

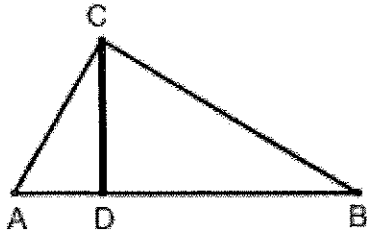
**2014 John O'Bryan Mathematical Competition**  
**Freshman-Sophomore Individual Test**

**Directions:** Please answer all questions on the answer sheet provided. All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. **Exact** answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. Set  $B = \left\{37, \sqrt{3}, -8, \frac{1}{2}, \pi\right\}$ . If one of the members of Set  $B$  is selected at random, find the probability that the member selected is an irrational number. Give your answer as a fully reduced fraction.
2. If  $x = 2.8$ , find the value of  $(3x + 7.6)^{-0.75}$ . Give your answer as a fully reduced fraction.
3. Suppose that  $C, D, E, F, G,$  and  $H$  are six distinct collinear points such that  $CD = \frac{1}{2}(DE) = EF = FG = GH$ . Further suppose that none of the segments  $CD, DE, EF, FG,$  or  $GH$  overlap except at shared end-points. If one of these six points is selected at random, find the probability that the point selected is one of the trisection points of the segment that joins  $C$  with  $H$ . Give your answer as a fully reduced fraction.
4. For all positive integers  $c$  and  $d$ , let  $c \oplus d = (c!)(2d) - (5d)$  and  $c \otimes d = d^2 - 3c^3$ . Find the value of  $(3 \oplus 6) - (2 \otimes 5)$ .
5. A rectangle has area 2014. The length of one side is 53. Find the perimeter of the rectangle.
6. If  $x - 12$  is a factor of  $x^2 - 2kx - 72$ , find the value of  $k$ .
7. A zookeeper was asked how many birds the zoo had in their flock. She answered: "If  $\frac{1}{3}, \frac{1}{5}, \frac{1}{7},$  and  $\frac{1}{8}$  of the flock were added together, they would total 1346." How many total birds are in the zoo's flock?
8. This question is multiple choice. For your answer, write the **capital letter** corresponding to the correct choice. The graph of  $y < 8x + 7$  is:
  - A. The interior of a circle
  - B. The region to the right of a vertical line.
  - C. The region below a horizontal line.
  - D. The region below a line with negative slope.
  - E. The region above a line with positive slope.
  - F. None of the choices (A–E) is correct.
9. Triangle  $ABC$  is isosceles with  $AB = AC = 15$  and  $BC = 24$ . Find the area of the triangle.
10. A rectangle has diagonal of length 17 and all sides are integers. From among all diagonals and sides of the rectangle, one segment is selected at random. Find the probability that it has length less than 11. Give your answer as a fully reduced fraction.
11. If a cleaning solution contains only alcohol and water, find the number of liters of water that must be added to reduce 9 liters of the cleaning solution containing 60% alcohol to a solution containing only 40% alcohol. Give your answer as an exact decimal.

12. A man has 30 coins in his pocket. The coins include only nickels, dimes, and quarters (there is at least one of each type). If the coins have a total value of \$5.10, find the maximum number of nickels the man can have in his pocket.

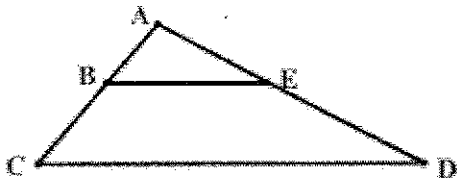
13. In the diagram,  $\overline{AC} \perp \overline{BC}$  and  $\overline{CD} \perp \overline{AB}$ . If  $AC = 3\sqrt{5}$  and  $BD = 12$ , find  $AB$ .



14. Find the value of  $k$  such that  $\frac{3}{k-2} = \frac{4}{k-3} - \frac{6}{k^2-5k+6}$ .

15. A line containing the point  $(-2, -5)$  also goes through the point  $(7, y)$ . The slope of the line is  $-\frac{2}{3}$ . Find the value of  $y$ .

16. Consider triangles  $ACD$  and  $ABE$  where  $\overline{BE} \parallel \overline{CD}$  as shown in the diagram. If  $AB = 13$ ,  $BC = 6$ ,  $AE = 12$ , and  $BE = 8$ , find 13 times the length of  $\overline{ED}$ .



17. Let  $m$  and  $n$  be positive integers such that  $m \neq n$ . It is known that  $\frac{1}{m}$  of the population of hippogriffs have long hair and horns while  $\frac{1}{n}$  of the population of hippogriffs have short hair and blue eyes. If these two groups comprise  $\frac{1}{93}$  of the entire population of hippogriffs, find the smallest possible value of  $(m+n)$ .

18. An escalator (going up) rises at a constant rate of  $k$  steps per second. Harry walks up the rising escalator by stepping up at a rate of 1 step per second and reaches the top in 44 seconds. Hermione runs up at a rate of 3 steps per second and reaches the top in 28 seconds. How many steps are on the escalator?

19. Find the smallest positive integer  $b$  such that the four numbers represented by  $\frac{b}{2}$ ,  $\frac{b+1}{3}$ ,  $\frac{b+2}{5}$ , and  $\frac{b+3}{7}$  are all integers.

20. A triangle has vertices at  $(6,7)$ ,  $(0,15)$ , and  $(2,4)$ . If the triangle is revolved around its shortest side, find the total surface area of the three-dimensional figure created by the rotation. Give your answer as a decimal rounded to the nearest tenth.

Name: \_\_\_\_\_

Team Code: \_\_\_\_\_

**2014 John O'Bryan Mathematical Competition  
Freshman/Sophomore Individual Test**

**Note: All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.**

1. \_\_\_\_\_

11. \_\_\_\_\_

2. \_\_\_\_\_

12. \_\_\_\_\_

3. \_\_\_\_\_

13. \_\_\_\_\_

4. \_\_\_\_\_

14. \_\_\_\_\_

5. \_\_\_\_\_

15. \_\_\_\_\_

6. \_\_\_\_\_

16. \_\_\_\_\_

7. \_\_\_\_\_

17. \_\_\_\_\_

8. \_\_\_\_\_

18. \_\_\_\_\_

9. \_\_\_\_\_

19. \_\_\_\_\_

10. \_\_\_\_\_

20. \_\_\_\_\_

Name: \_\_\_\_\_ **ANSWERS** \_\_\_\_\_

Team Code: \_\_\_\_\_

**2014 John O'Bryan Mathematical Competition  
Freshman-Sophomore Individual Test**

**Note:** All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).

1.        $\frac{2}{5}$        Must be in  
this form

11.       4.5       Must be in  
this form

2.        $\frac{1}{8}$        Must be in  
this form

12.       9      

3.        $\frac{1}{6}$        Must be in  
this form

13.       15      

4.       41      

14.       5      

5.       182      

15.       -11      

6.       3      

16.       72      

7.       1680      

17.       496      

8.       F       Must be this  
capital letter.

18.       154      

9.       108      

19.       158      

10.        $\frac{1}{3}$        Must be in  
this form

20.       665.4       Must be in  
this form

Awards Lists and Solutions to the Team Competition may be found at  
<http://math.nku.edu/job>